2.2 Region Growing and Merging

The goal of region merging and region growing [16], [26] is to divide the domain $R$ of the image $I$ into regions \( \{R_i : i = 1, ..., M \} \) so that \( R = \bigcup_{i=1}^{M} R_i, \quad R_i \cap R_j = \emptyset \) if \( i \neq j \), and $I$ satisfies a homogeneity criterion on each $R_i$.

Region merging builds up complicated regions by combining smaller regions using a statistical similarity test. A popular choice is Fisher's test [33]. For example, suppose there are two adjacent regions $R_1$ and $R_2$, where $n_1$, $n_2$, $\hat{\mu}_1$, $\hat{\mu}_2$, $\hat{\sigma}_1^2$, $\hat{\sigma}_2^2$ are the sizes, sample means, and sample variances of $R_1$, $R_2$, respectively. Then in order to decide whether or not to merge them, we can look at the squared Fisher distance:

\[
\frac{(n_1 + n_2)(\hat{\mu}_1 - \hat{\mu}_2)^2}{n_1 \hat{\sigma}_1^2 + n_2 \hat{\sigma}_2^2} = \frac{n \hat{\sigma}^2}{n_1 \hat{\sigma}_1^2 + n_2 \hat{\sigma}_2^2} - 1, \tag{4}
\]

where $n = n_1 + n_2$ and $\hat{\sigma}^2$ is the sample variance of the mixture region (a generalization to the multidimensional case is called Hotelling's test [17]). If this statistic is below a certain threshold then the regions are merged.

Region growing can be considered as a special case of region merging, where $R_1$ is the growing region and $R_2$ is a single pixel at the boundary of $R_1$, i.e., $n_2 = 1$ and $n_1$ is very large (say $n_1 > 100$). In this case we can treat $\mu = \hat{\mu}_1$, $\sigma^2 = \hat{\sigma}_1^2$, and $\mu_2 = I_{(x,y)}$ (the intensity at point $(x, y)$), and approximate the squared Fisher distance [8] by: $\frac{(I - \mu)^2}{\sigma^2}$.