EN250 - Medical Imaging Analysis

Lab # 2: CT Simulation & Reconstruction - Part 2

Due: Thursday, Feb 25, 2011

CT Reconstruction: Filtered Back Projection

This lab is a continuation of Lab 1. We will reconstruct an image from its Radon transform using the Filtered Back Projection technique and compare the results with those using simple back projection. Let $\mu(i,j)$ be an image and $\rho_\phi(i')$ be its Radon transform. We will filter the Radon transform $\rho_\phi(i')$ to generate $\rho^*_\phi(i')$ before applying back projection to reconstruct the original image:

$$\rho^*_\phi(i') = h(i') * \rho_\phi(i'), \tag{1}$$

where $*$ is the convolution operation. Some of the popular filters are Ram-Lak, Shepp-Logan, Hamming window, and Rosenfield-Kak. In the discrete domain, these filters take the form

- Ram-Lak Filter

$$h[k] = \begin{cases} \frac{\pi}{4}, & \text{if k=0} \\ -\frac{1}{\pi k^2}, & \text{if k odd} \\ 0, & \text{if even} \end{cases} \tag{2}$$

- Shepp-Logan Filter

$$h[k] = \frac{2}{\pi(4k^2-1)}, \quad \text{k} = 0, \pm 1, \pm 2, \ldots \tag{3}$$

- Hamming-window

$$h[k] = 0.53836 - 0.46164 \cos\left(\frac{2\pi k}{N-1}\right), \tag{4}$$

  where N is the width, in number of samples, of the window function.

- Rosenfield-Kak, continuous form:

$$h(x) = \frac{\epsilon^2 - (2\pi x)^2}{[\epsilon^2 + (2\pi x)^2]^2} \tag{5}$$

  The discrete form is obtained by sampling the continuous form. See page 17 of the "CT notes" for a description of this filter.

1. Using the phantom image computed in Problem 3 of Lab 1 as test image:

a. Compute its Radon transform.

b. Reconstruct the original image using filtered back projection. Show results for all 4 types of filters described above.

c. Compare the results with the reconstruction using simple back projection.

2. Repeat the steps in Problem 1 for the following images:

a. The synthetic images generated in Problem 1 of Lab 1 (filled circle, filled rectangle, and filled triangle).

b. "CT-image.tif"

c. The Shepp-Logan phantom generated using Matlab.
Command: `I = phantom('Modified Shepp-Logan',256);`